

Testing the symmetrization postulate of quantum mechanics and the spin-statistics connection

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Abstract

Recent experimental tests of the symmetrization postulate of quantum mechanics are discussed. It is shown that in a strict sense these experiments cannot test the validity of the symmetrization postulate, but in most cases do test the spin-statistics connection. An experiment is proposed that would allow to search for possible violations of the symmetrization postulate.

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I. INTRODUCTION

Several experiments have been reported that assert to test the symmetrization postulate of quantum mechanics and/or the spin-statistics connection. The symmetrization postulate and the spin-statistics connection are indeed amongst the fundamental tenets of quantum mechanics. The symmetrization postulate establishes that in a system containing identical particles the only possible states are either all symmetrical or all antisymmetrical with respect to permutations of the particles. In the first case, the particles are called bosons and follow Bose-Einstein statistics; in the second case they are called fermions and follow Fermi-Dirac statistics. Experiments indicate that particles with integer values of spin are bosons, while particles with odd-half-integer spin are fermions. The reason why only symmetric and antisymmetric states seem to occur in nature and the connection with the spin of the particles has been a puzzle since the early days of quantum mechanics [1]. The spin-statistics theorem, proved by W. Pauli [2] from the basic principles of quantum field theory and special relativity such as the requirement of local commutativity of observables, states that, given the choice between Bose and Fermi statistics, integer-spin particles must obey Bose statistics and odd-half-integer-spin particles must obey Fermi statistics. Proofs of the spin-statistics theorem are reviewed in [3].

Quantum mechanics would nevertheless allow also symmetries different from those imposed by the symmetrization postulate, and theories have been developed allowing for small deviations from conventional statistics which might have been masked in the experiments performed so far. It is worth noting that no theory so far predicts the possibility of observing a violation in a particular system or in some specific condition. Consistent theories can be formulated, however, which would lead to different symmetry properties. Experiments are needed then to discriminate between these theories imposing constraints which are the more stringent the higher is the experimental precision.

In this paper, after a brief account of the theoretical background, a discussion of experimental tests is presented, trying to clarify in a consistent manner their meaning. It is shown

that most of the recent experiments should be considered as testing the spin-statistics connection rather than the validity of the symmetrization postulate. An experiment to search for violations of the symmetrization postulate for nuclei is proposed.

II. THEORETICAL BACKGROUND

It is not the purpose of this paper to discuss in detail the theories which lead to symmetry properties different from the ones which are peculiar to bosons and fermions. A general survey can be found in [4]. It is of interest, however, to put in evidence the possibility of theories allowing for small deviations from the usual symmetry relations, whose search is the subject of the experiments discussed in this paper. Such deviations can be expressed as a different symmetry of the state under particle exchange or, in Fock-space representation, as a deformation of the algebra of the creation and annihilation operators.

A statistics intermediate between Bose and Fermi cases was first proposed in [5] considering the possibility that at most n identical particles could occupy the same quantum state. This idea led to a generalized field theory [6], called parastatistics, in which the field operators obey trilinear commutation relations instead of the usual bilinear relations. These theories predict, however, gross violations of statistics which are immediately excluded by experimental evidence.

The possibility of a continuous interpolation between bosonic and fermionic behaviours is given by "quons" [7]. The commutation and anticommutation relations are replaced by generalized bilinear commutation relations depending on a parameter q (q -mutators):

$$a_k a_l^+ - q a_l^+ a_k = \delta_{kl}, \quad -1 \leq q \leq 1 \quad (1)$$

with the vacuum condition $a_k|0\rangle = 0$. As q varies between -1 and 1, the symmetry changes continuously from the completely antisymmetric case (fermions) to the completely symmetric case (bosons). It has been shown that this interpolation preserves positivity of norms and the non-relativistic form of locality [8]. Other aspects are still doubtful such as the possibility of accounting for local observables in a relativistic theory or for the existence of antiparticles.

Statistics other than Fermi and Bose have also been investigated for one- and two-dimensional systems [9] and in connection with anyon high-temperature superconducting systems [10].

Two important points must be noted to avoid confusion in the interpretation of experimental tests. The first is that in a system including two identical particles, only two "entangled" states are possible, namely a symmetric and an antisymmetric state:

$$\begin{aligned}\Psi_S(1, 2) &= \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2) + \psi_b(1)\psi_a(2)] \\ \Psi_A(1, 2) &= \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2) - \psi_b(1)\psi_a(2)]\end{aligned}\tag{2}$$

In this case, the symmetrization postulate does not play any role, the question being only the connection between the spin of the particles and the symmetry of the two-particle state. This is not the case for systems including more than two identical particles, where mixed-symmetry states are possible. The second important remark is that, as was pointed out in [11], in the framework of ordinary quantum mechanics transitions between states of different permutation symmetry cannot take place. This "superselection rule" is a rigorous selection rule holding also in the presence of perturbations such as collisions or external fields. It is not possible then to consider states given by a coherent superposition of states of different permutation symmetry. The system must be described as an incoherent mixture which is represented by a density matrix. In the case of two particles, for example, the two-particles density matrix for small violations of Bose statistics is:

$$\rho_2 = (1 - \frac{1}{2}\beta^2)\rho_s + \frac{1}{2}\beta^2\rho_a.\tag{3}$$

where $\rho_{s(a)}$ is the symmetric (antisymmetric) two-particle density matrix. Neglecting these important points led to a wrong interpretation of some of the experiments.

Using the notation adopted in the literature, in the following $\beta^2/2$ indicates the "symmetry-violation" parameter. Its real meaning needs to be specified, however, for the particular physical system and the theoretical model considered. In [7], for example, it was

shown that in the frame of a q -mutator theory, the value of $\beta^2/2$ can be related to the value of the q parameter.

III. EXPERIMENTS

In this section, experiments performed to search for violations of the symmetrization postulate and/or of the spin-statistics connection are discussed. The focus is on recent experiments and, in particular, on experiments on integer-spin nuclei in molecules. For a review of the work before 1989, the reader is referred to [12]. As already mentioned, some of the initial experiments suffered from a misunderstanding of the constraints imposed by the "superselection rule". It is shown here that also the interpretation given for some recent experiments is not correct. In fact, although the papers published on this subject usually present their "null results" as a confirmation of the validity of the symmetrization postulate, most of them should be considered as tests of the spin-statistics connection. The argument is simple: as discussed above, in a two-particle system, only symmetric or antisymmetric states are possible. The symmetrization postulate does not play any role in this case. Experiments involving only two identical particles should then be considered as tests of the spin-statistics connection rather than tests of the symmetrization postulate. An experiment that would allow a genuine test of the symmetrization postulate is proposed in the following.

A. Experiments on half-integer-spin particles.

A few experiments have been performed to test the validity of the Pauli principle for half-integer-spin particles. In particular, a high precision test on electrons was performed in [13] by running a current through a copper bar and searching for the X-rays that would be emitted if some of the electrons introduced in the sample were captured by a copper atom and cascaded down to the 1S state, which is already filled with two electrons. No signal was found and this was interpreted as giving a limit $\beta^2/2 \leq 1.7 \times 10^{-26}$ to the probability that a new electron introduced into copper would form a mixed-symmetry state with respect to

the other electrons already present in the copper sample. The large number of electrons in the system was important to reach such a high sensitivity but, on the other hand, makes the interpretation of the result more complicated. Conclusions may change depending on whether we consider the symmetry of the system composed by the injected electron plus the electrons already present in the copper bar, or we consider a model in which the electron collides with a copper atom and is captured.

A simpler two-electron system was investigated in [14]. A spectroscopic test was performed on helium atoms, searching for a transition involving the permutation symmetric $1s2s\ ^1S_0$ state. An upper bound $\beta^2/2 \leq 5 \times 10^{-6}$ was set to a violation of the Pauli principle. In spite of the lower sensitivity, the interpretation of this result is simpler. Since only two identical particles are involved, this must be considered as a test of the spin-statistics connection. Doubts can be raised, however, about what would be the chemical stability of "paronic" atoms in ordinary samples. In [14], this was taken into account by having the atoms ionize and recombine in a discharge before entering the detection region.

B. Experiments on integer-spin particles.

No accurate test for integer-spin particles had been reported until recently. This is due to the fact that while there are several systems in which a violation of the Pauli exclusion principle would be detected as a signal on a zero background, the effect of a small violation for particles following Bose-Einstein statistics would usually manifest itself as a small change in the properties of a many-particle system. This obviously limits the achievable accuracy. In [12], a bound to a possible violation of the generalized Bose statistics for pions was inferred considering the $K_L \rightarrow \pi^+\pi^-$ decay, which is usually considered as due to CP violation. A limit of $\beta^2/2 \leq 10^{-6}$ was obtained.

Photons.

Several papers have been published recently reporting or proposing experiments to set a

limit to possible violations of Bose statistics for photons [15–20]. The fundamental nature of the photon and its peculiar properties makes it very interesting to investigate this particle in this context. It is hard, however, to find an experiment that would give a direct evidence of a violation with a significant sensitivity. This is one case, in fact, in which a small deviation from normal statistics would usually produce only a small signal over a large background. An attempt to set a limit to a possible violation of Bose statistics was made in [15], based on light intensities attainable in laser systems. In [16], a possible dependence of the frequency of light on its intensity was searched for. This effect is expected if a q -nonlinearity is introduced in the description of the electromagnetic field. Since nonlinearities in the commutation relations give rise to mixed-symmetry states, this experiment could also be reinterpreted as a search for a violation of the symmetrization postulate for photons. The connection is not straightforward though and was not pursued in the paper.

In [17], the experimental upper limit on the two-gamma decay of Z -boson, $Z \rightarrow \gamma\gamma$, was used to establish an upper bound $\beta^2/2 \leq 10^{-2}$ on a possible small violation of the exchange-symmetry for a system of two photons. The same idea, based on what is called Landau-Yang theorem, was exploited in [18] to improve the limit by searching for the forbidden $J = 0 \rightarrow J' = 1$ transitions in atoms excited by two photons of the same energy. A limit of $\beta^2/2 \leq 7 \times 10^{-8}$ was set on the probability that two photons are in an exchange-antisymmetric state. A different approach was followed in [20]. A very tight bound to a violation of statistics for photons was inferred considering photons and electrons as coupled "quons" and relating the bound for photons to that obtained in [13] for electrons. Although this argument is indirect and model-dependent, it is very interesting and it could also be extended to other particles.

Nuclei in molecules.

In 1931, Ehrenfest and Oppenheimer showed that a composite system of fermions is a boson or a fermion depending on whether it is made of an even or an odd number of fermions [21]. Considering the total angular momentum resulting from the constituents angular

momenta, an extension of the spin-statistics theorem to composite systems is obtained. For this argument to be valid, it is necessary that the interaction between the composite particles is negligible compared to the internal excitation energy so that the internal structure can be neglected and the system can be considered as a single particle. This has a striking demonstration in recent experiments on Bose-Einstein condensation of atoms [22]: the phase transition is observed only for the isotopes of the atoms for which the sum of the number of protons, neutrons and electrons gives an even number, that is when the number of neutrons in the nucleus is even. The same argument applies to the symmetry of systems including identical nuclei. In [23,24], it was proposed that a high-sensitivity investigation of the spectra of molecules containing identical spin-0 nuclei would allow to search for violations of Bose statistics for the nuclei. The basic idea of these experiments is analogous to the one underlying the experiment on electrons in the helium atom [14]. This represents indeed a rare case in which a violation of the spin-statistics connection for integer-spin particles would be detected on a virtually zero background with a sensitivity comparable to the one achieved in experiments on fermions. Let us consider a molecule containing two identical spin-0 nuclei as, for example, $^{16}\text{O}_2$. According to the Born-Oppenheimer approximation and neglecting the coupling of the nuclear spin with the rest of the molecule (which is not important for these experiments since the spin of the nuclei is zero), the total wave function ψ_t can be written in the form

$$\psi_t = \psi_e \psi_v \psi_r \psi_n \quad (4)$$

where ψ_e , ψ_v and ψ_r are the electronic, vibrational and rotational functions, respectively, and ψ_n is the nuclear spin function. For integer-spin nuclei, the total wavefunction ψ_t must be symmetric in the exchange of two nuclei. Since the nuclear spin is zero, ψ_n is obviously symmetric. The vibrational wave function ψ_v is also unaltered in the exchange of the nuclei because it depends only on the magnitude of the internuclear distance. Since the total wavefunction ψ_t must be symmetric, only the states corresponding to even (odd) rotational quantum numbers are allowed if ψ_e is symmetric (antisymmetric) [25].

In a first series of experiments [26–28], the spectrum of the $^{16}\text{O}_2$ molecule was investigated searching for transitions between states which are antisymmetric under the exchange of the two nuclei. An upper bound $\beta^2/2 \leq 5 \times 10^{-7}$ was set in [27], and a similar, slightly less accurate, result was obtained in [28]. Experiments on $^{16}\text{O}_2$ were later reported in [29,30]. In [30], a limit $\beta^2/2 \leq 5 \times 10^{-8}$ was obtained.

In [31] a new accurate test on ^{16}O nuclei was performed by investigating the vibrational spectrum of the $^{12}\text{C}^{16}\text{O}_2$ molecule. The CO_2 molecule has the same symmetry properties of O_2 but, since it is triatomic, it has strong active vibrational bands in the infrared which are lacking in O_2 spectra. In particular, the intensity of the combination band $12^0_1 - 00^0_0$ around $2\text{ }\mu\text{m}$ investigated in [31] is more than two orders of magnitude larger than the electronic transitions of oxygen previously investigated, and therefore the absorbance of a given population is correspondingly larger. This results in an increased detection sensitivity. Since the ground electronic wavefunction of the CO_2 molecule is symmetric in the exchange of the two ^{16}O nuclei, the rotational wavefunction in the ground vibrational state must be symmetric and only even values for the rotational quantum number J are allowed. The R-branch investigated in [31] should then be composed only of $\text{R}(2J)$ transitions. A bound of $\beta^2/2 \leq (2.1 \pm 0.7) \times 10^{-9}$ to the relative population of the forbidden states was deduced in this work. This experiment gives at present the most stringent test of the spin-statistics connection for ^{16}O nuclei. Further improvements are possible: the sensitivity could be increased by reducing the noise to the quantum level, by increasing the absorption pathlength, or by selecting stronger transitions. The fundamental vibrational band of CO_2 around $4.3\text{ }\mu\text{m}$, which is at least a factor of thousand stronger than the one at $2\text{ }\mu\text{m}$, could be investigated. A dramatic increase in the detection sensitivity could be obtained by a resonant-ionization-spectroscopy detection scheme. A similar test can be performed on other spin-0 nuclei; a straightforward extension is the investigation of the spectrum of $^{18}\text{O}_2$ [32].

An interesting prospect is the investigation of spectra of molecules containing more than two identical nuclei. As shown above, most of the experiments performed so far involve only two identical particles so that they provide a test of the spin-statistics connection. In order

to search for possible violations of the symmetrization postulate of quantum mechanics, it is now clear that systems including more than two identical particles should be considered. In this case, indeed, more complex symmetries are possible and the Young diagrams do not reduce to the trivial completely symmetric and completely antisymmetric cases. On the other hand, systems composed of very large numbers of identical particles can complicate the interpretation of experimental results. Molecules offer the possibility of investigating more and more complex structures and, as for the experiments performed on O_2 and CO_2 , high detection sensitivity can be achieved by laser spectroscopy methods. A good candidate for this experiment is OsO_4 , a highly symmetric molecule with four identical spin-0 nuclei. In spite of the higher complexity of the spectrum with respect to the simpler molecules investigated previously, high resolution and high sensitivity spectroscopy schemes have been developed, especially in the region around $10\ \mu m$ which is of metrological interest. The good knowledge of molecular parameters makes it simpler to find the position of the relevant transitions and to separate them from spurious signals. In particular, transition frequencies can be singled out that would represent a signature of a violation of the symmetrization postulate. The expected detection sensitivity is expected to be comparable to that achieved in previous tests on molecules.

IV. CONCLUSIONS

Several experiments confirm the validity of the spin-statistics connection for various types of particles to a high level of accuracy. This provides a proof of the general formalism of quantum mechanics and can be extended also to different particles. In a recent paper [33], the results obtained in [31] for nuclei was used to set a bound on possible violations of the Pauli exclusion principle for nucleons and for quarks. On the other hand, not many experiments have really tested the validity of the symmetrization postulate of quantum mechanics with high precision. A possible experiment on nuclei in molecules was proposed in this paper which should allow to test the symmetrization postulate with an accuracy comparable to

that achieved in previous tests of the spin-statistics connection.

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